Orthogonality Preserved – v5

Efficient and accurate bin-center population for seismic data loading

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Agenda

- Definitions and scope
  - Orthogonal, conformal, re-projection
- Conformal projections preserve orthogonality
  - Cartographic examples
  - TM to TM
- Trigonometric bin-center population
  - Whole-project "best-fit" parallelogram induces error and non-orthogonality
  - Segmented-project quadrilateral gridding is accurate, orthogonal and fast

Orthogonal Defined

- “At right angles. For instance, two curves are said to be orthogonal if their tangents at the point of intersection are perpendicular”
- The Penguin Dictionary of Mathematics, Penguin Books, London, 1989

Statement of the Issue

- Seismic projects acquired and/or processed orthogonally in one projection are often worked/displayed in a different projection
- Trigonometric population of bin centers in an interpretation workstation can induce non-orthogonally and inaccuracy
- Segmenting the job (gridding) can maintain efficiency and improve accuracy

Conformality Defined

- "The property of transformation from one surface to another which insures that the angle between any two curves on one surface is preserved in magnitude and sense by the angle between the corresponding curves on the other surface"
- Geodetic Glossary, National Geodetic Survey, September, 1986

Another Definition of Conformality

- "Many of the most common and most important map projections are *conformal* or *orthomorphic*..., in that normally the shape of every *small* feature of the map is shown correctly... An important result of conformality is that relative angles at each point are correct, and the local scale in every direction around any one point is constant."
- John P. Snyder, Map Projections Used by the U. S. Geological Survey, 1983

Our Scope

- Conformal projections are those constructed mathematically to preserve real-world angles
- <u>This presentation deals with conformal projections only</u>
- Non-conformal projections must be handled differently
- <u>Datum shifting is not addressed in this presentation</u>

Reprojection Defined

- The cartographic process of converting E/N in one projection (or grid, i.e. P1) to Lat/Lon first and then to E/N in a different projection (or grid, i.e. P2)
  - E/N (P1) => LL (ellipsoid)
  - LL (ellipsoid) => E/N (P2)
- Reprojection from one <u>conformal</u> projection to another also preserves angles

Conformal Reprojection

- Reprojection from one conformal projection to another also preserves angles
- Syllogistically, we deduce that
  - If $\angle P1 = \angle LL$
  - and $\angle LL = \angle P2$
  - then $\angle P2 = \angle P1$
- Conformal reprojection preserves orthogonality
- Also preserves shape over a small area
  - Certainly an area the size of a seismic bin

More on Re-projection

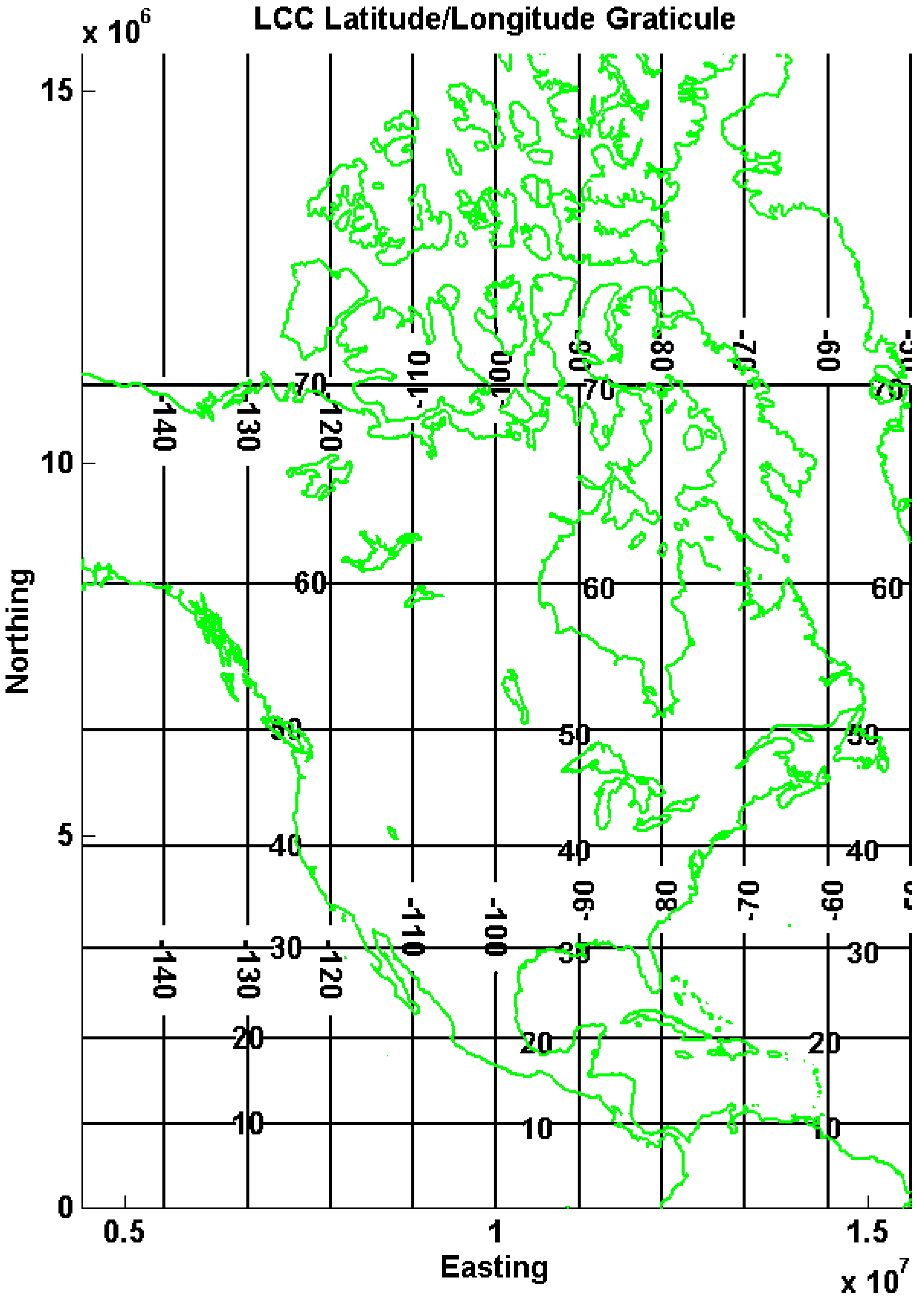
- Changes shape slightly over a large area
  - For example, a square project of several hundred kilometers may become "trapezoidal"
- Tends to curve lines (that were straight in the old projection) in the new projection
  - Dependent upon the projections involved
  - Intersecting angles are preserved
- An orthogonal seismic bin is orthogonal in any conformal projection

Cartographic Examples

- Straight lines can intersect orthogonally
- Curved lines can intersect orthogonally
- Mercator Projection
  - Straight meridians and straight parallels
- Lambert Conformal Conic projection
  - Straight meridians and curved parallels
- Transverse Mercator projection
  - Curved meridians and curved meridians

Orthogonal Meridians and Parallels

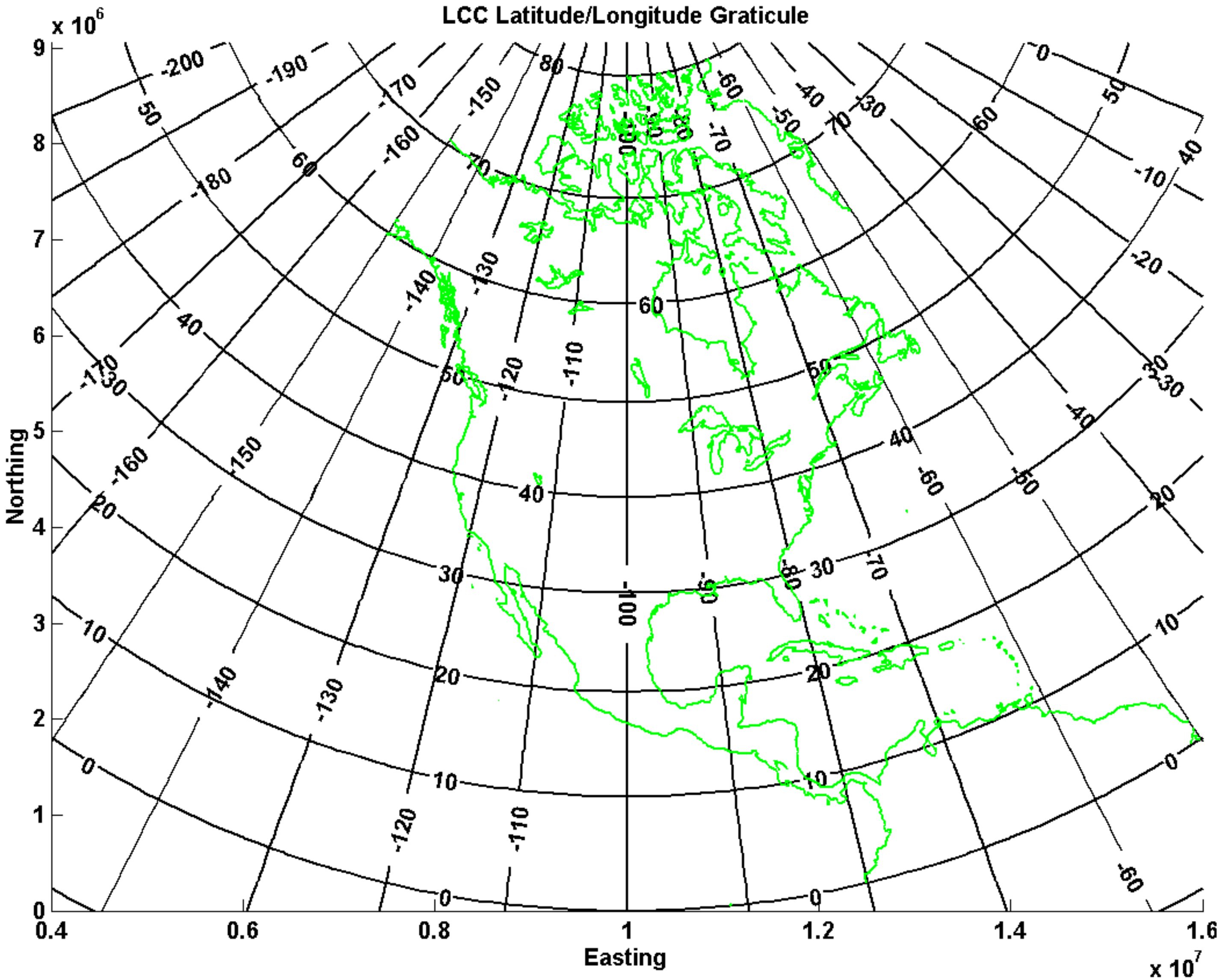
On the ellipsoid curved meridians and curved parallels intersect orthogonally



Mercator Projection

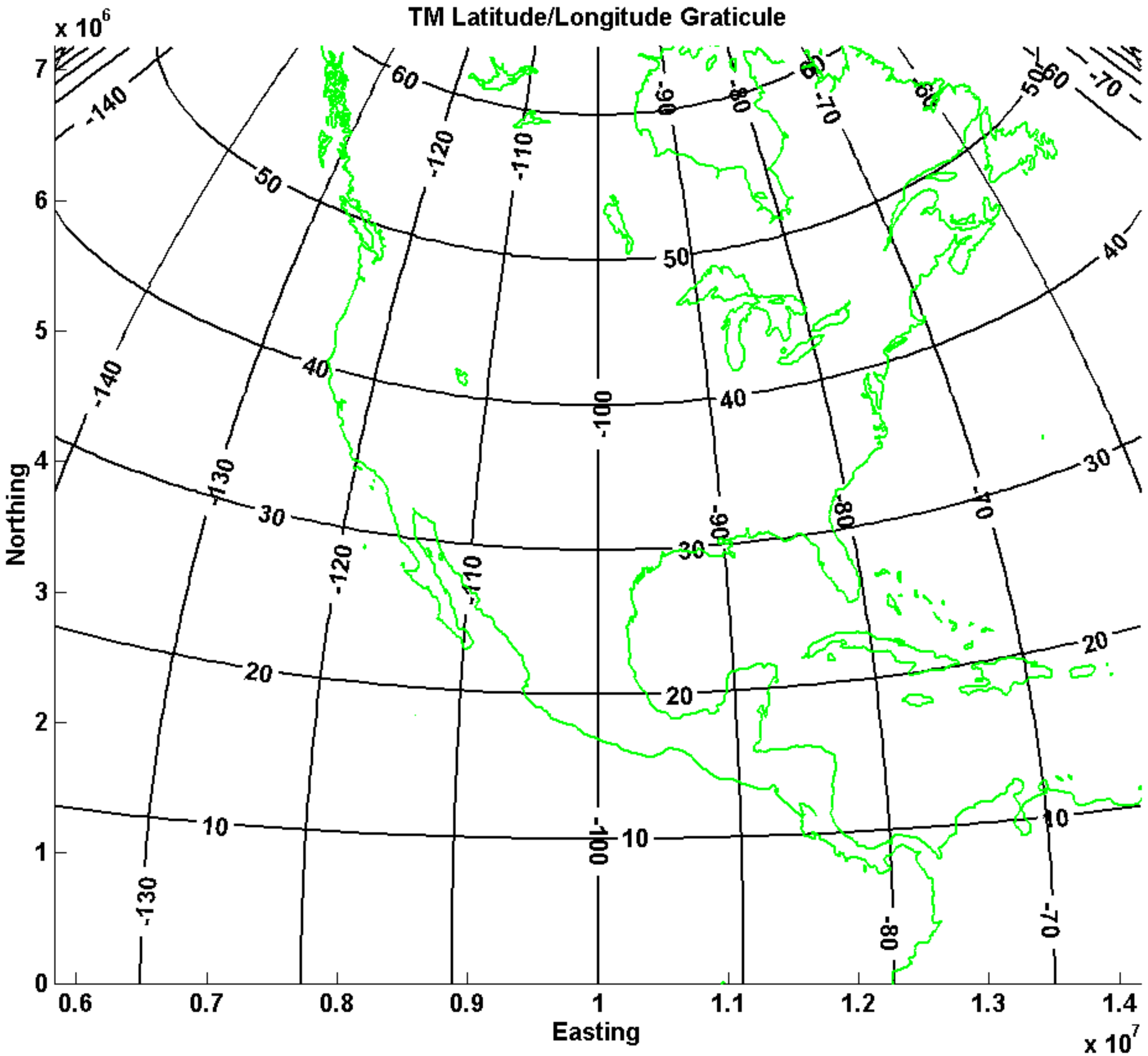
Meridians and parallels are straight lines, which intersect orthogonally.

Lambert Conformal Conic Projection



Meridians are straight lines and parallels are curved. They intersect orthogonally.

Transverse Mercator Projection

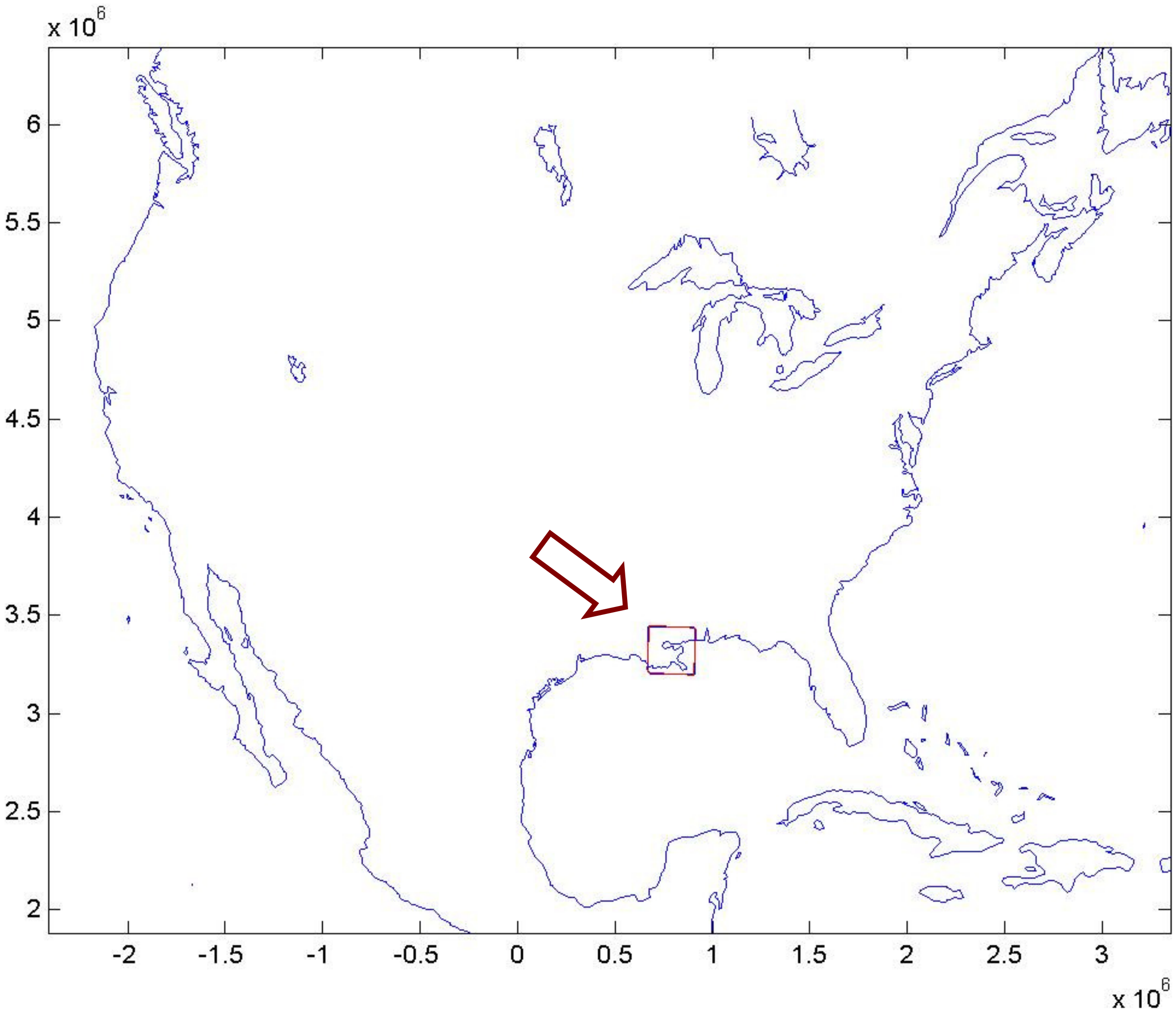


Meridians and parallels are curved lines, which intersect orthogonally.

TM to TM

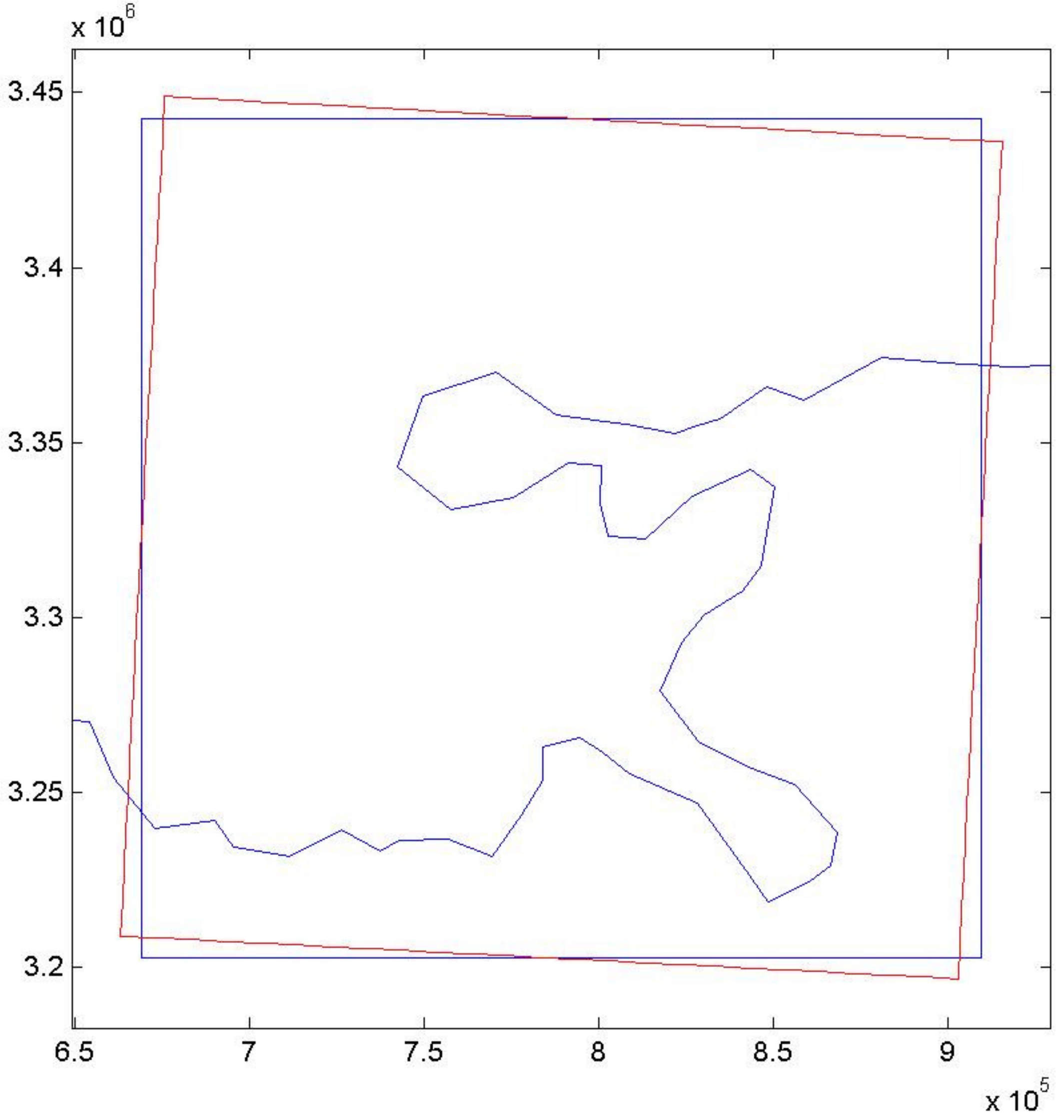
- A square 240km by 240km project in UTM Zone 15 is cartographically re-projected into UTM Zone 14
- The Zone 14 project center is translated over the center of the Zone 15 project center
- The orientation difference is due to convergence of the meridians
- The projects are rotated into best alignment
- Close-up views exhibit curvature

Area of Interest (AOI)



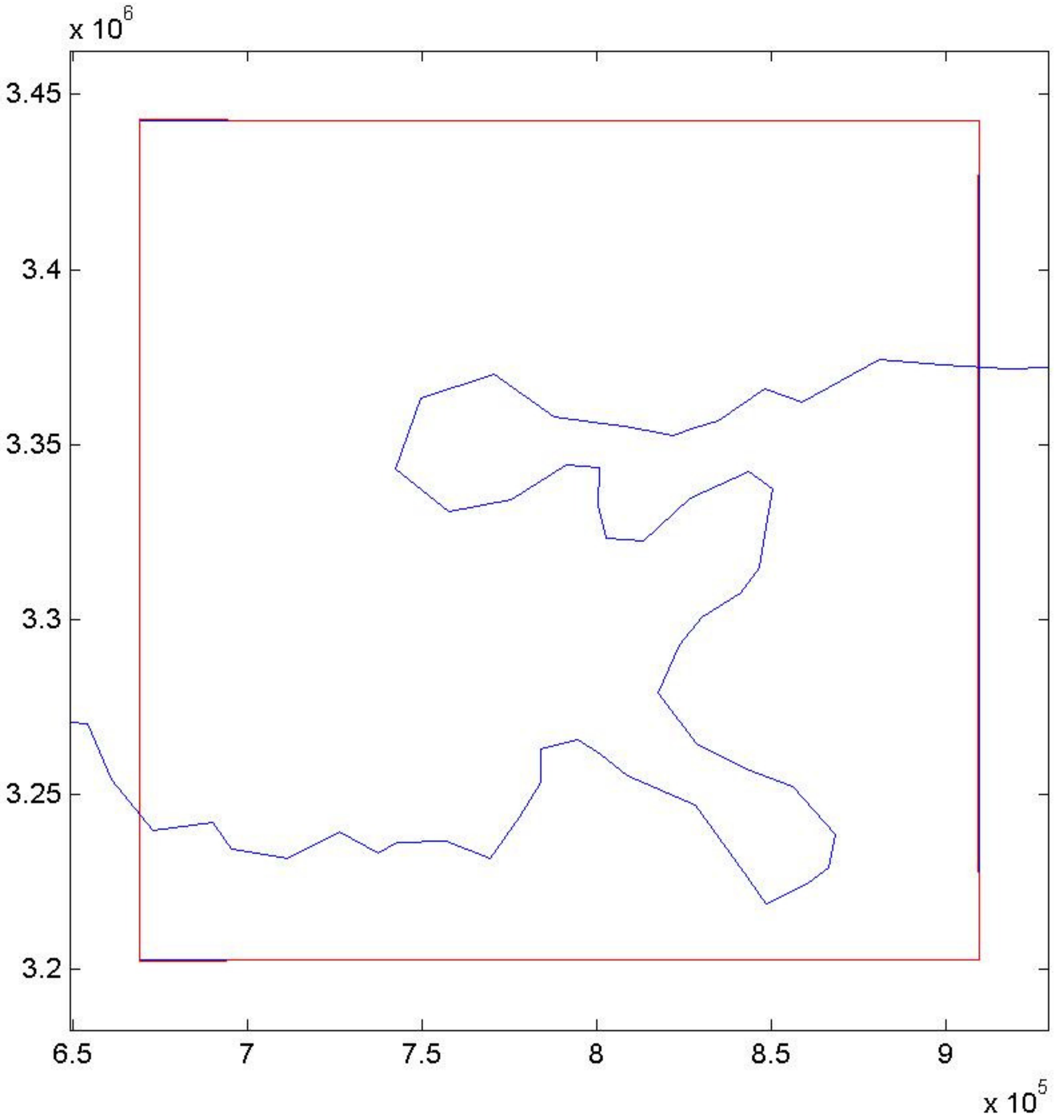
The AOI is centered at 30N / 90W. It extends 120km to the N, E, S and W, i.e., a perimeter with sides of 240km. The center (90W) divides UTM Zones 14 and 15.

Translated Perimeter



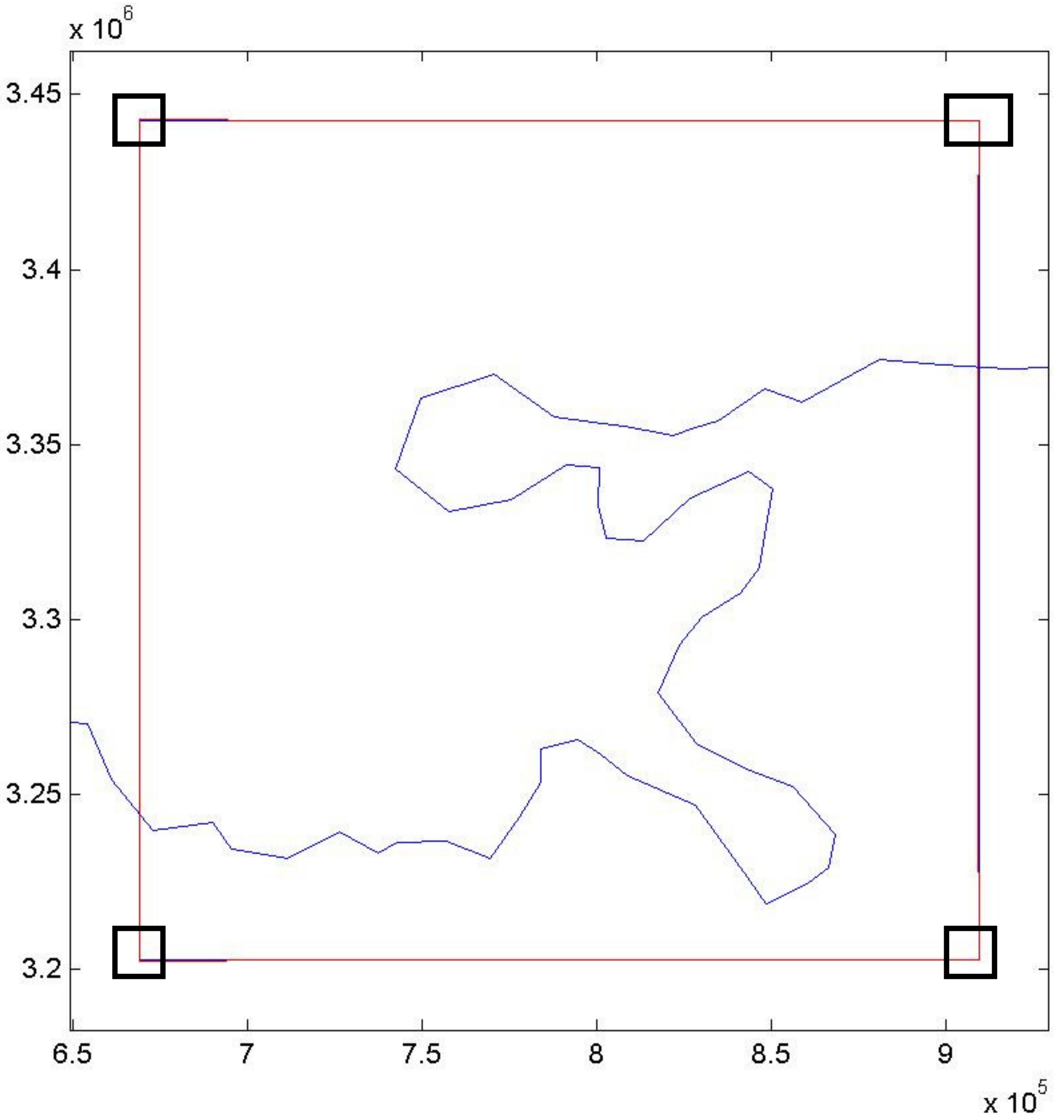
The blue (square) perimeter is the original Zone 15 AOI. The red perimeter is the AOI re-projected into Zone 14 then translated so that the centers of the two perimeters coincide. The difference in orientation is due to convergence of the meridians.

Aligned Perimeter



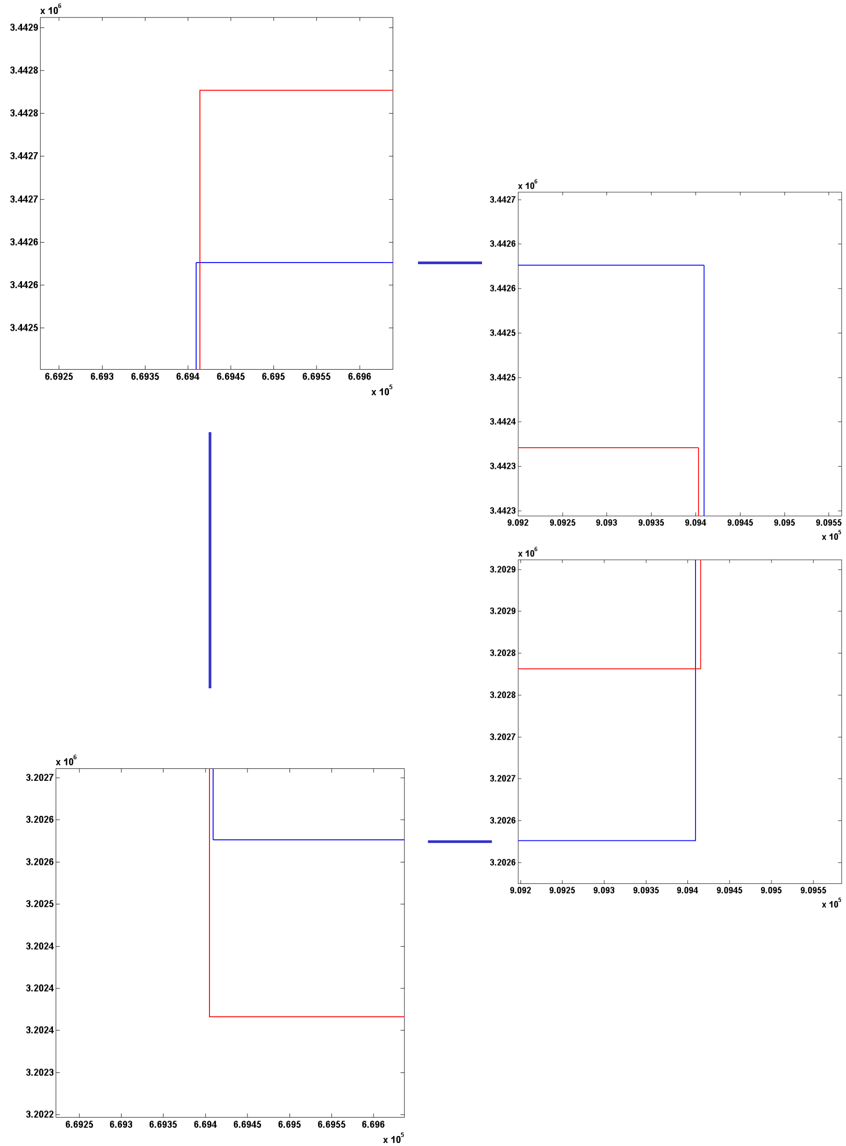
In this slide the re-projected Z14 red perimeter is rotated into best alignment with the original Z15 blue perimeter.

The Corners



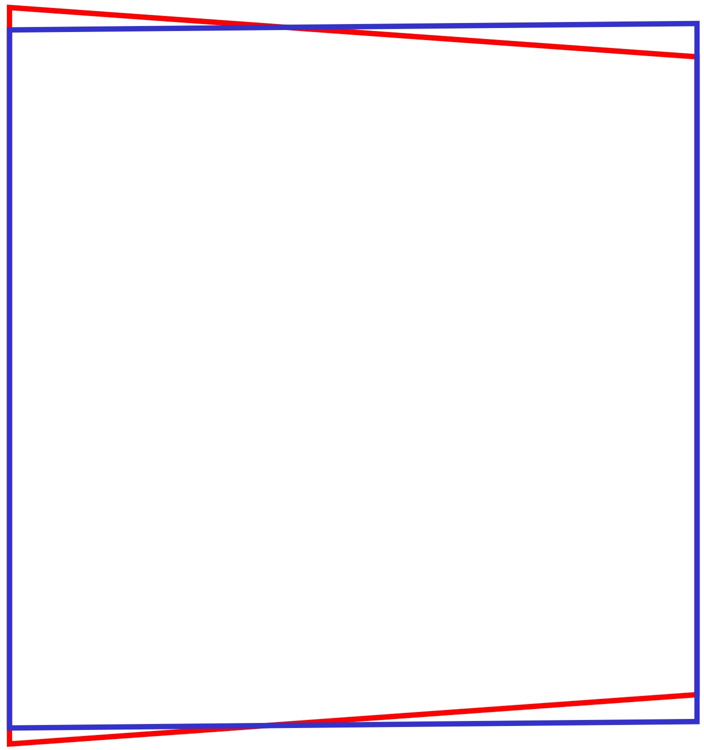
In the next slide we'll view only the four corners shown as small squares to better determine the shapes of the blue and red perimeters.

Corners Exhibiting Quadrilateral



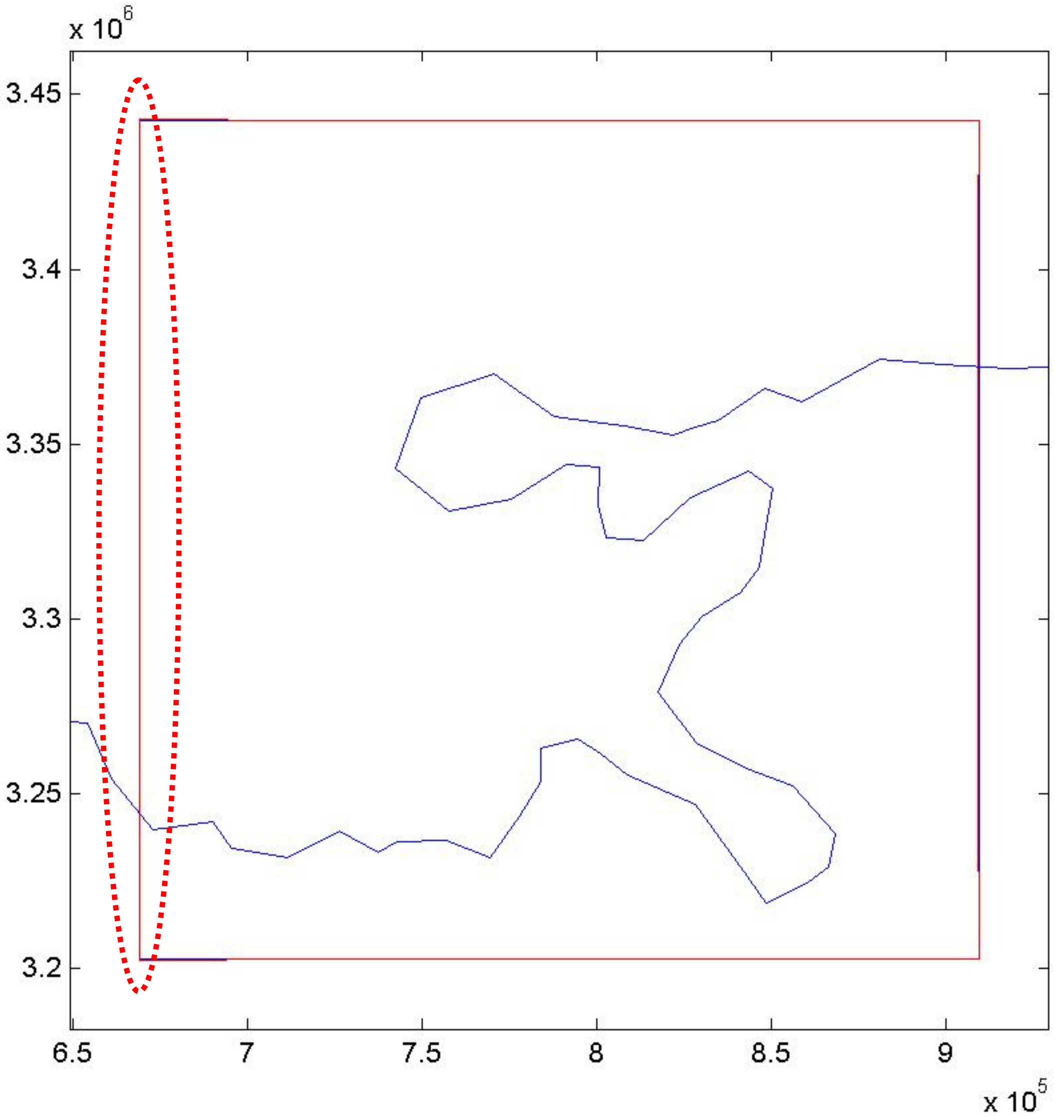
This slide exhibits close-up views of the four corners of the superimposed perimeters. The "trapezoidal" shape of the re-projected red Z14 perimeter is clearly seen. This is due to the varying point scale factor over the zone. All corners (blue and red) are orthogonal.

The Perimeters Superimposed



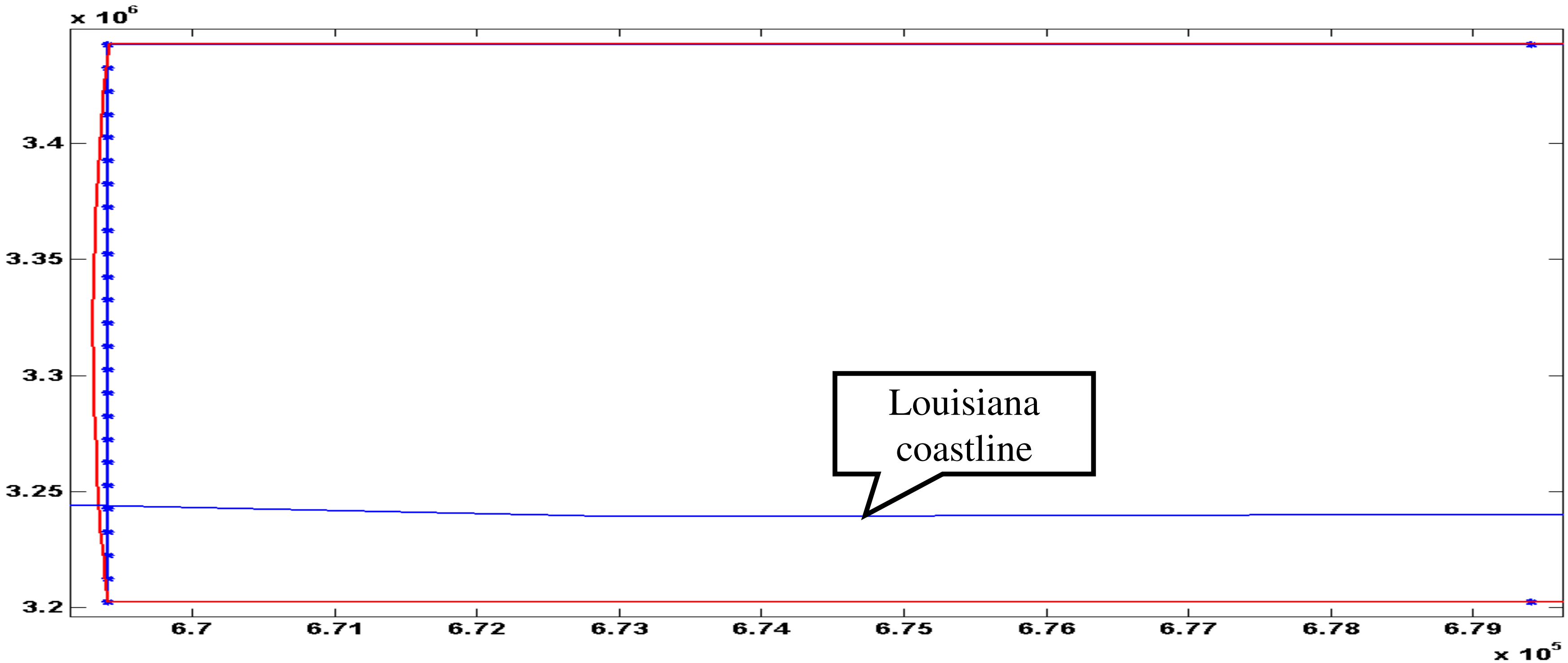
This cartoon exhibits in exaggeration what we saw on the previous slide. How is orthogonality preserved in the red perimeter?

Focus on the Western Side



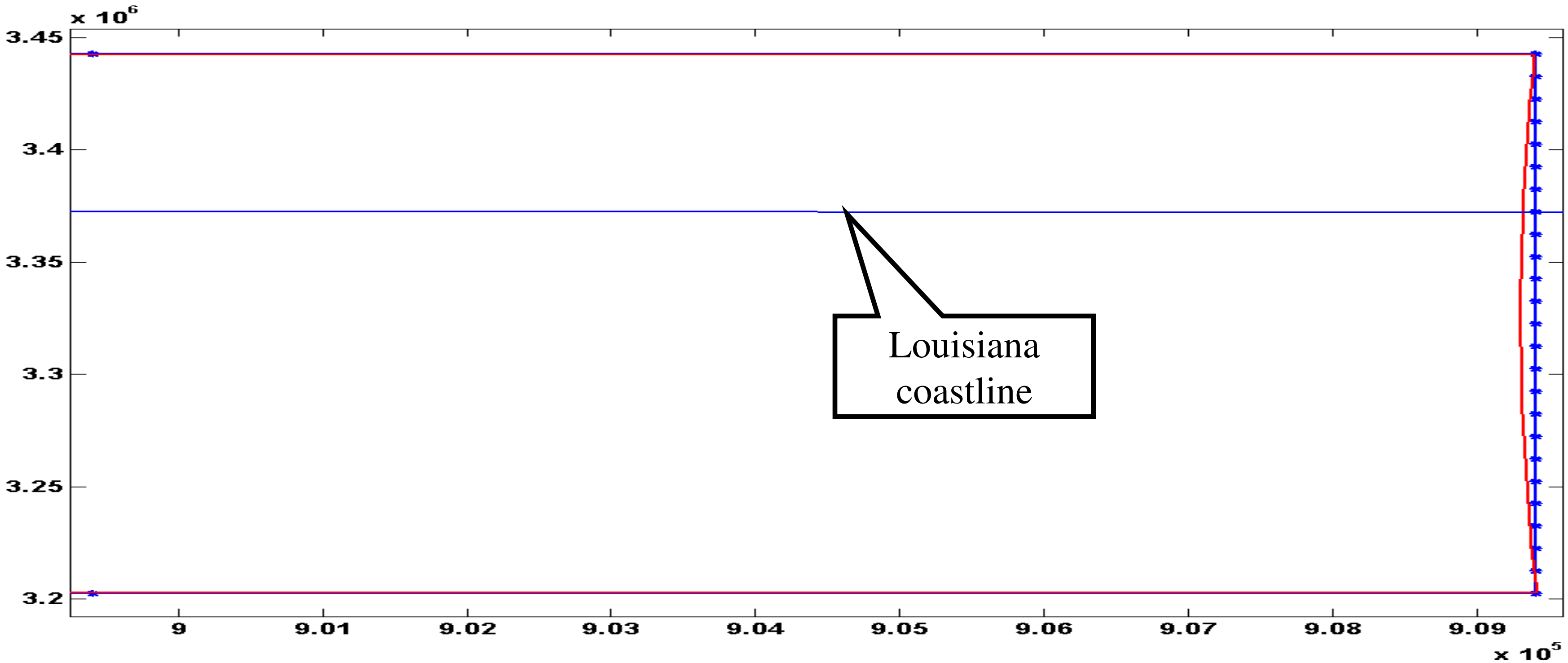
The next side will show just that part of the perimeter within the dashed red ellipse, in extreme aspect ratio.

Orthogonality Preserved - West



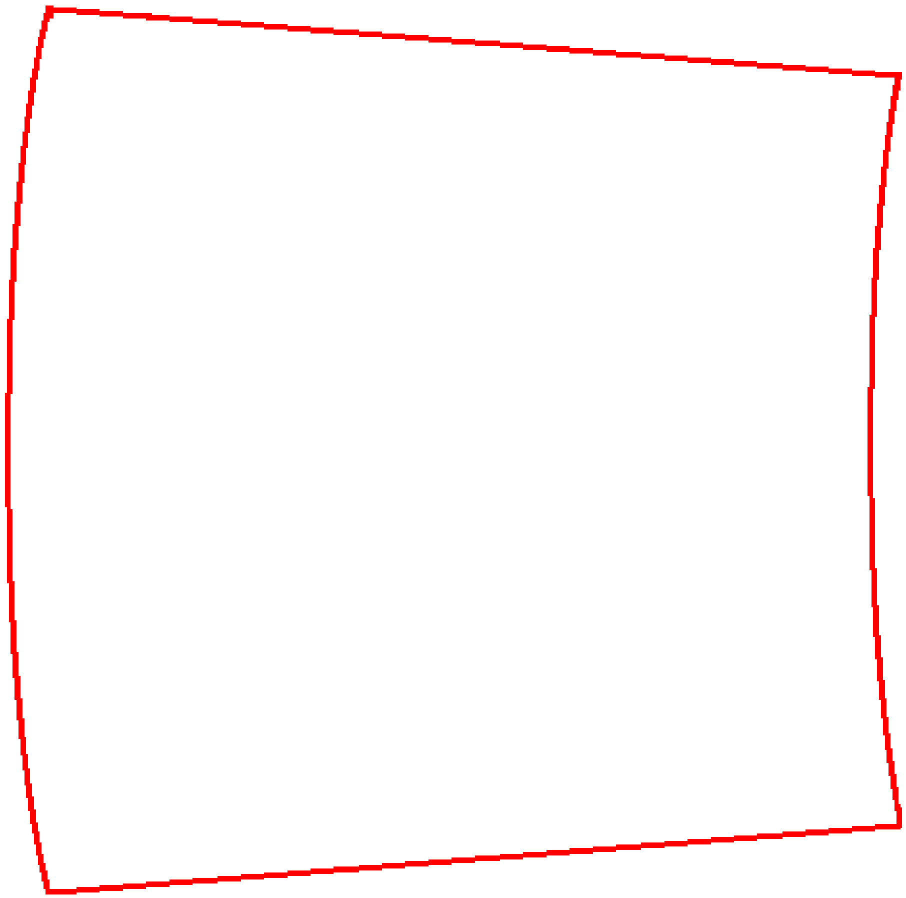
This slide exhibits the western boundary of the two perimeters in extreme aspect ratio. Asterisks are placed on the blue perimeter every 10km. The graphic is 240km high and 10km wide. Notice the curvature of the red, reprojected Z14 boundary. This curvature preserves orthogonality at the corners.

Orthogonality Preserved - East



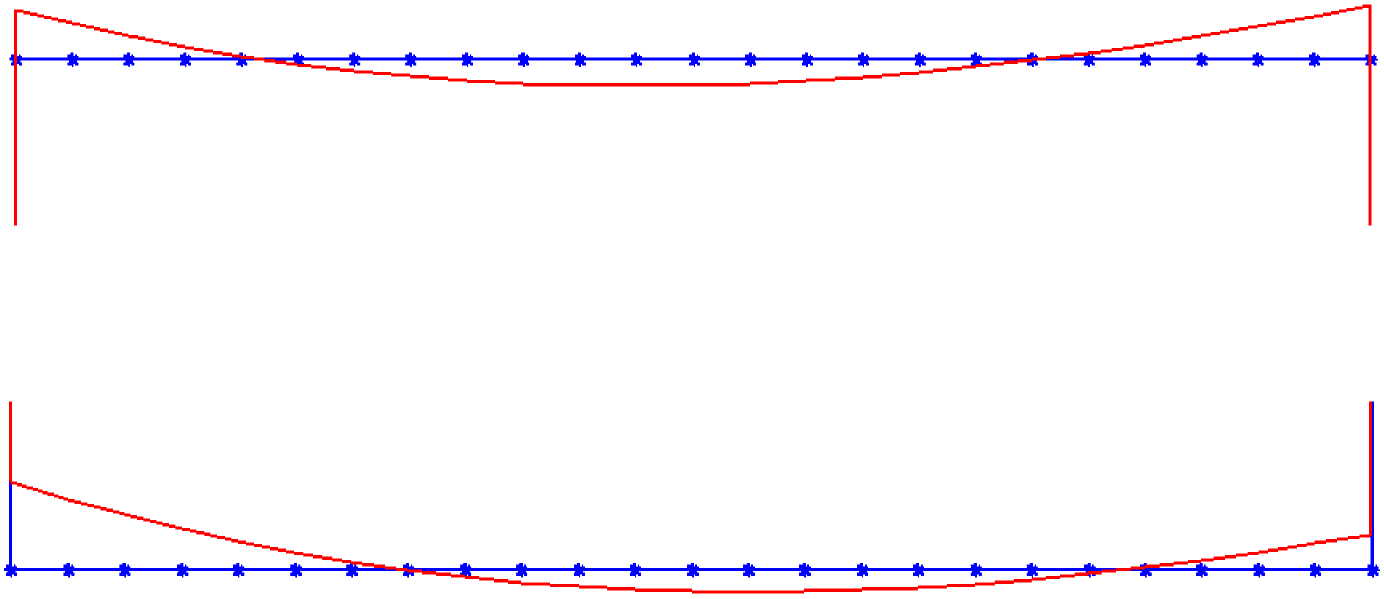
This slide exhibits the <u>eastern boundary</u> of the two perimeters in extreme aspect ratio. Asterisks are placed on the blue perimeter every 10km. The graphic is 240km high and 10km wide. Notice the curvature of the red, re-projected Z14 boundary. This curvature preserves orthogonality at the corners.

The Re-projected Perimeter

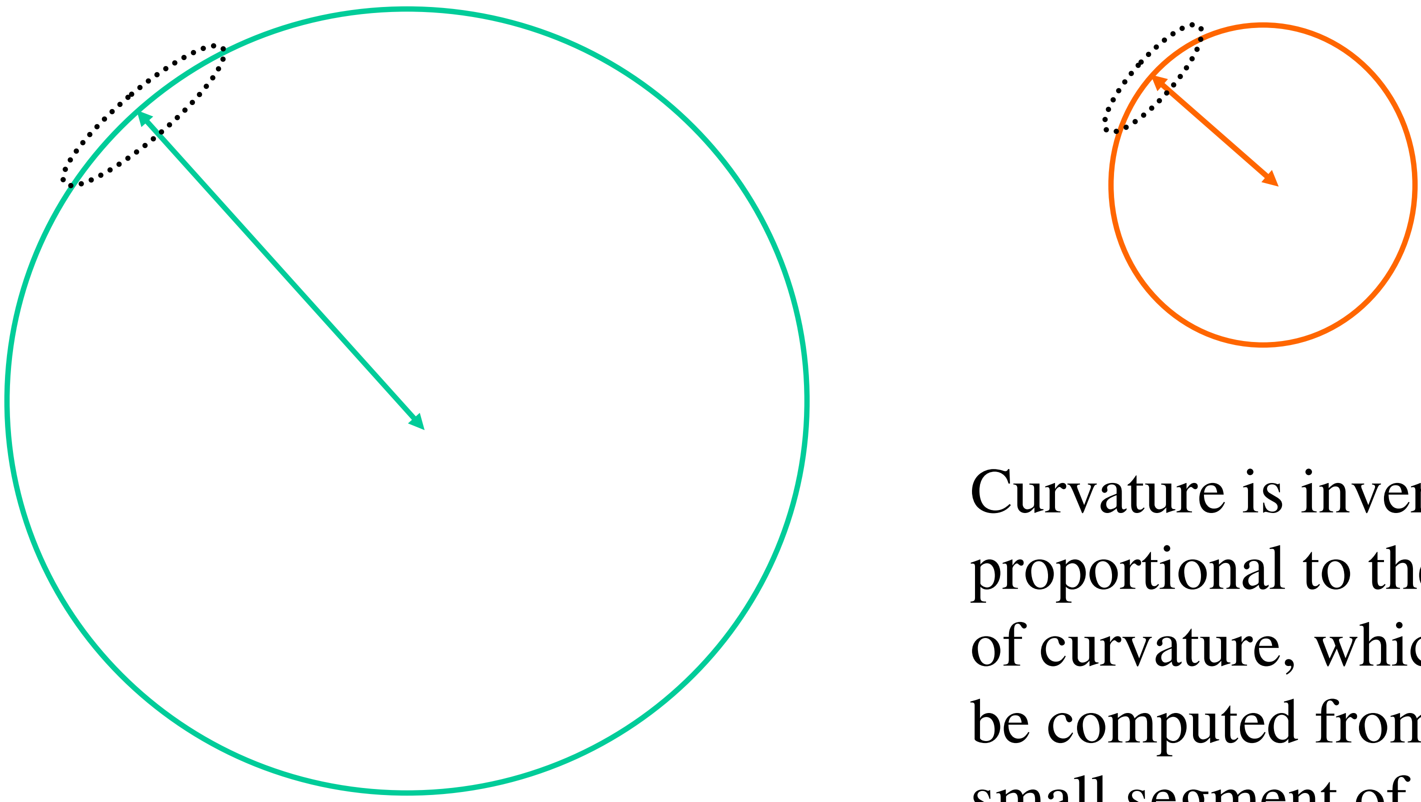


The greater curvature of the East and West boundaries (compared to the North and South boundaries) maintain orthogonality at the corners of the “quadrilateral”, i.e. all right angles at the vertices.

Orthogonality Preserved
North and South Boundaries
Reframed in Very Extreme Aspect

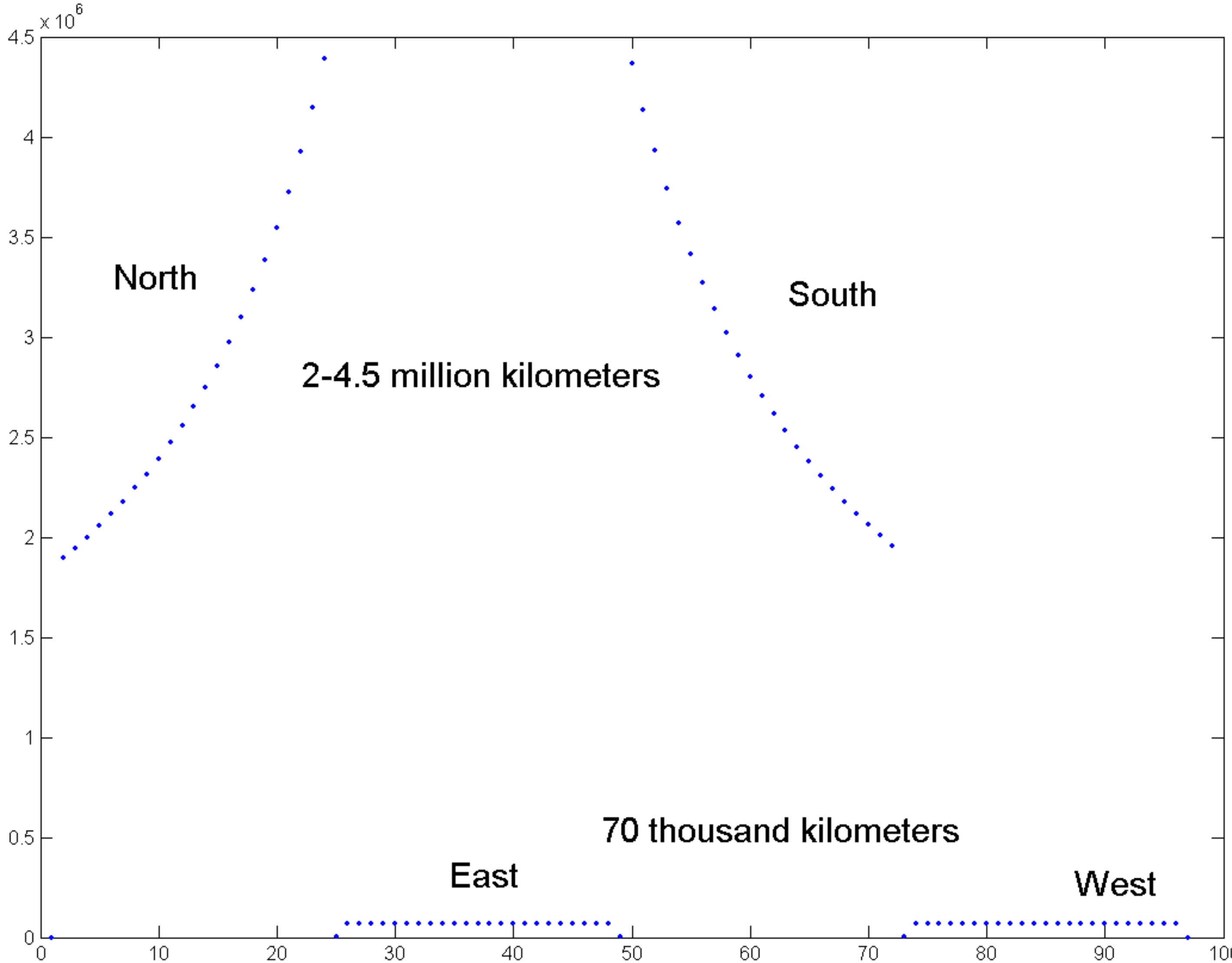


Radius of Curvature

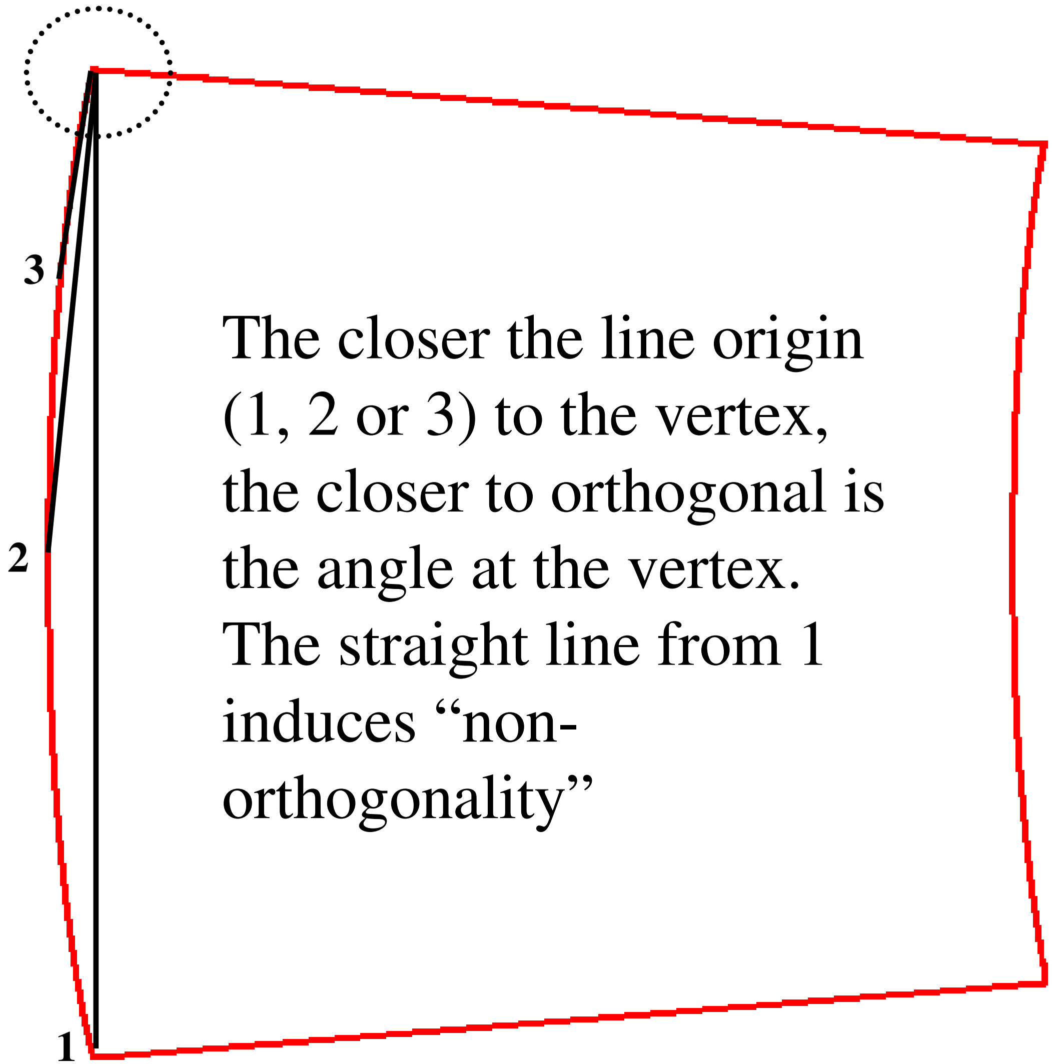


Curvature is inversely proportional to the radius of curvature, which can be computed from a small segment of the curve.

Radii of Curvature Every 10km

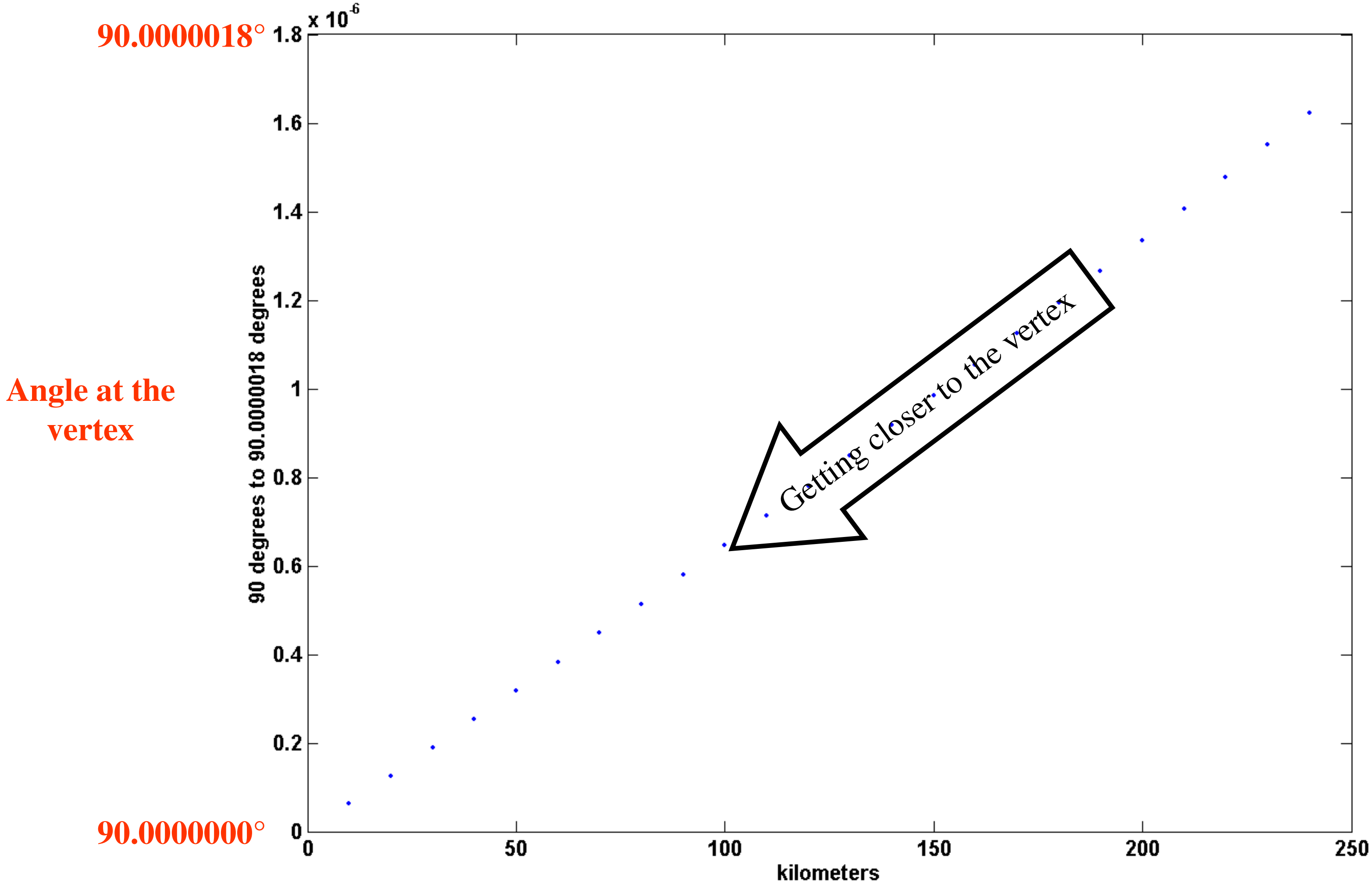


Angle at the Corner

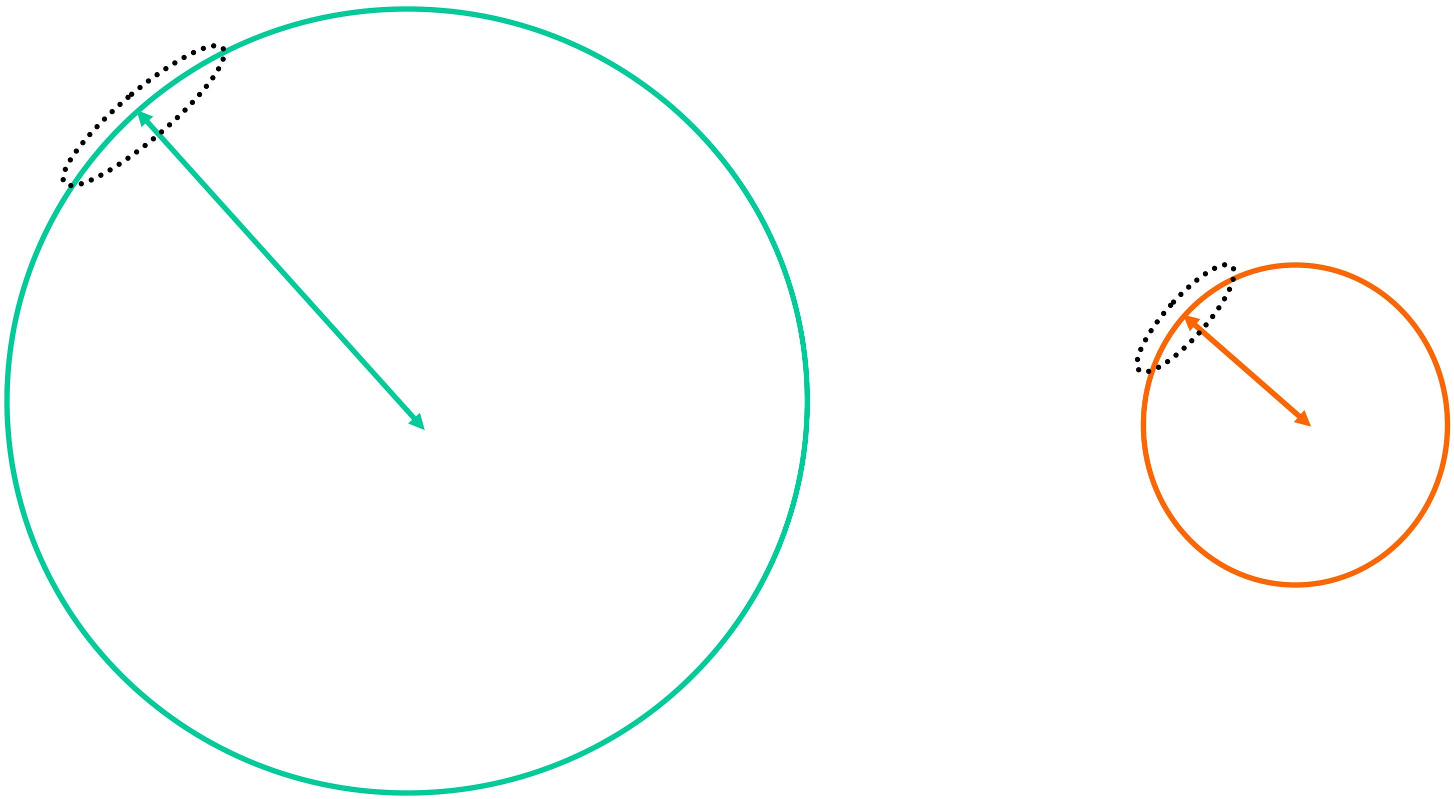


The closer the line origin (1, 2 or 3) to the vertex, the closer to orthogonal is the angle at the vertex. The straight line from 1 induces "non-orthogonality"

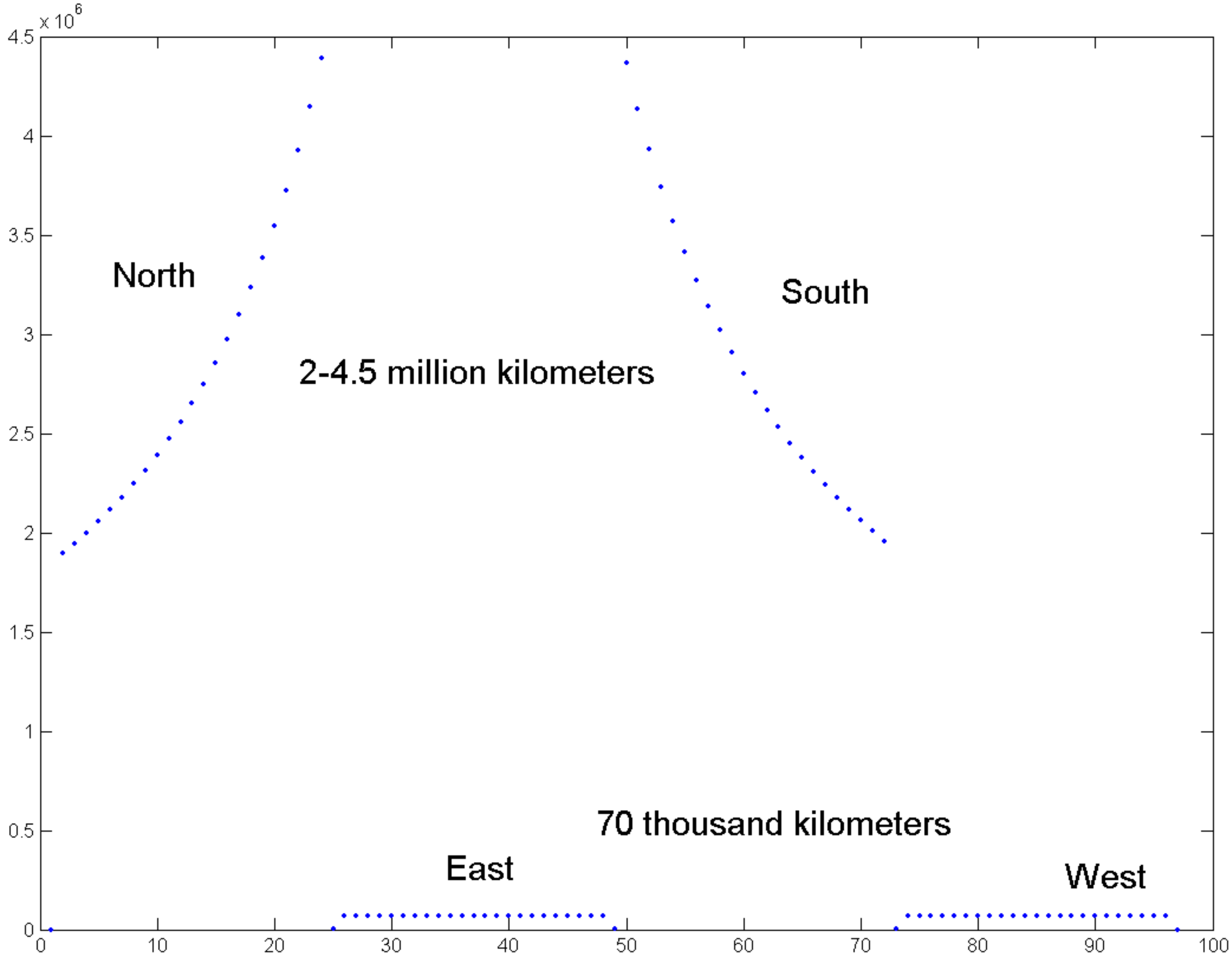
Corner Angle as Function of Distance every 10km



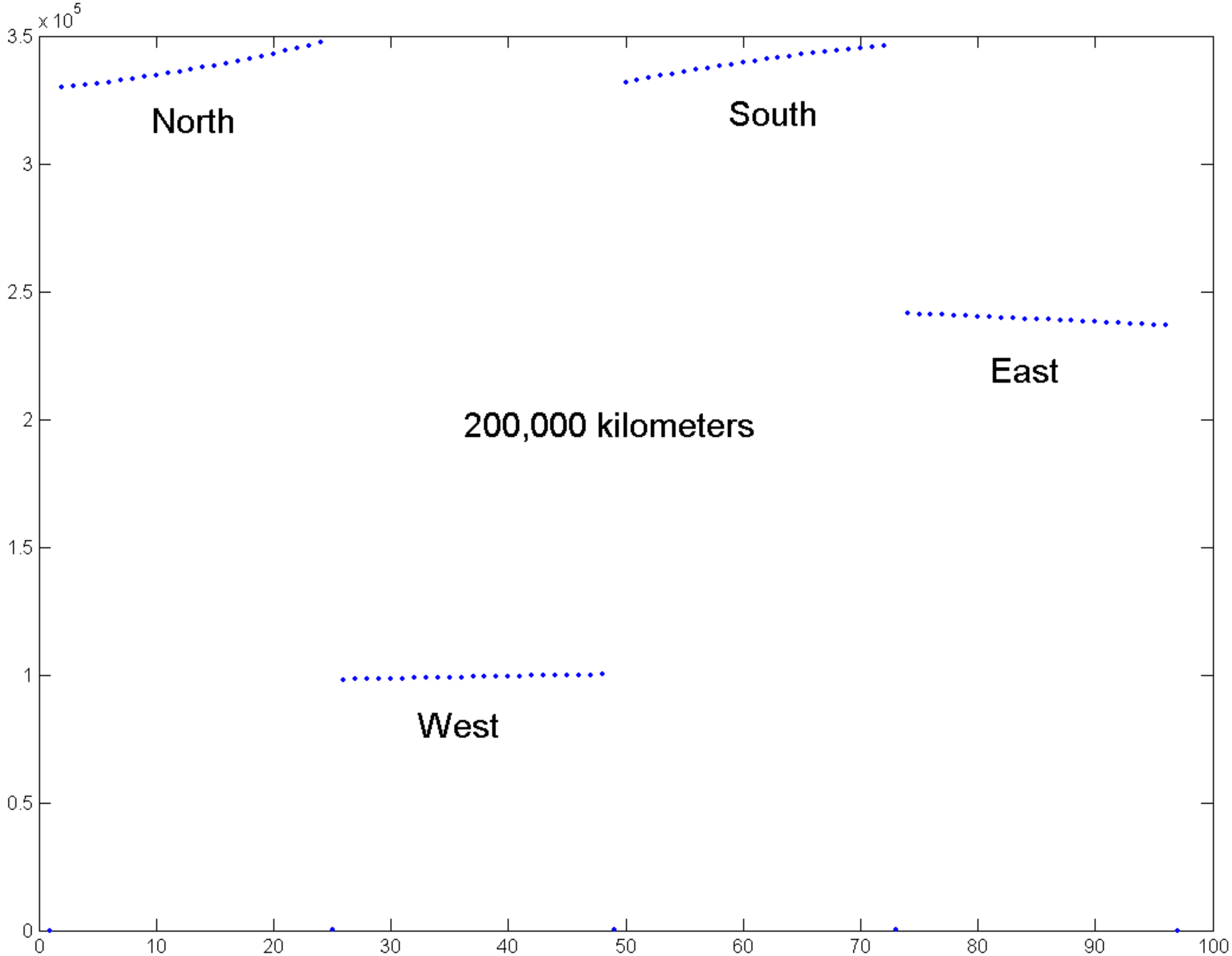
Radii of Curvature



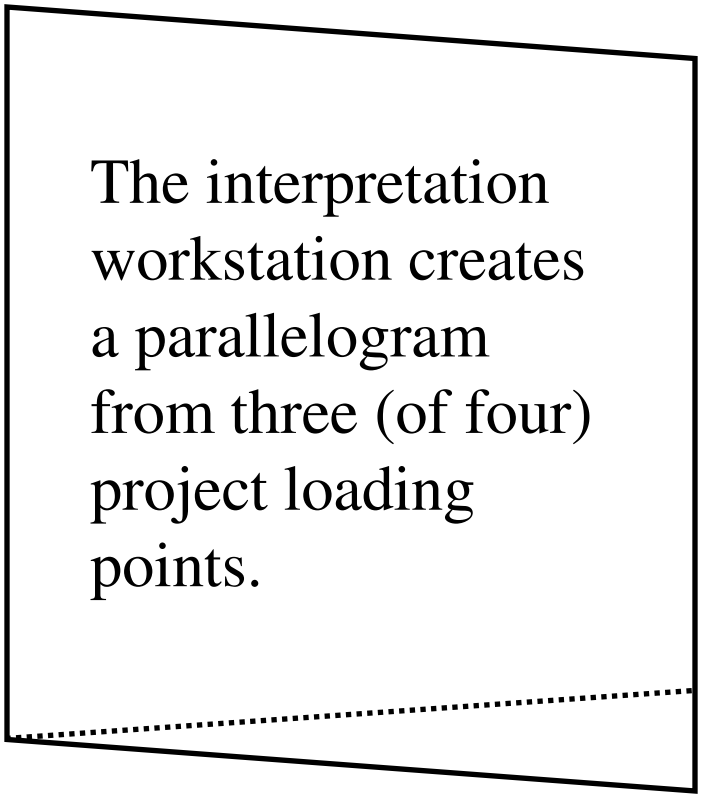
Radii of Curvature: TM to TM

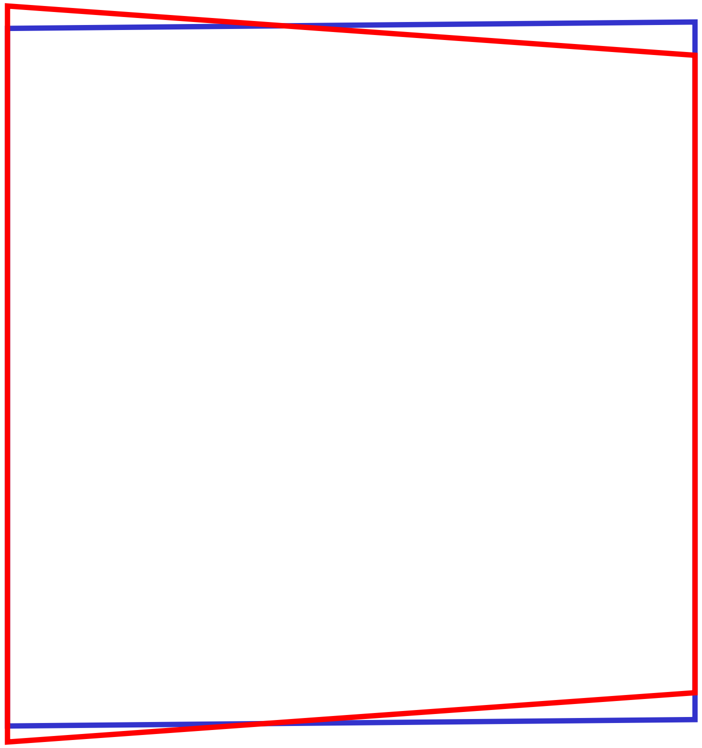


Radii of Curvature: TM to LCC

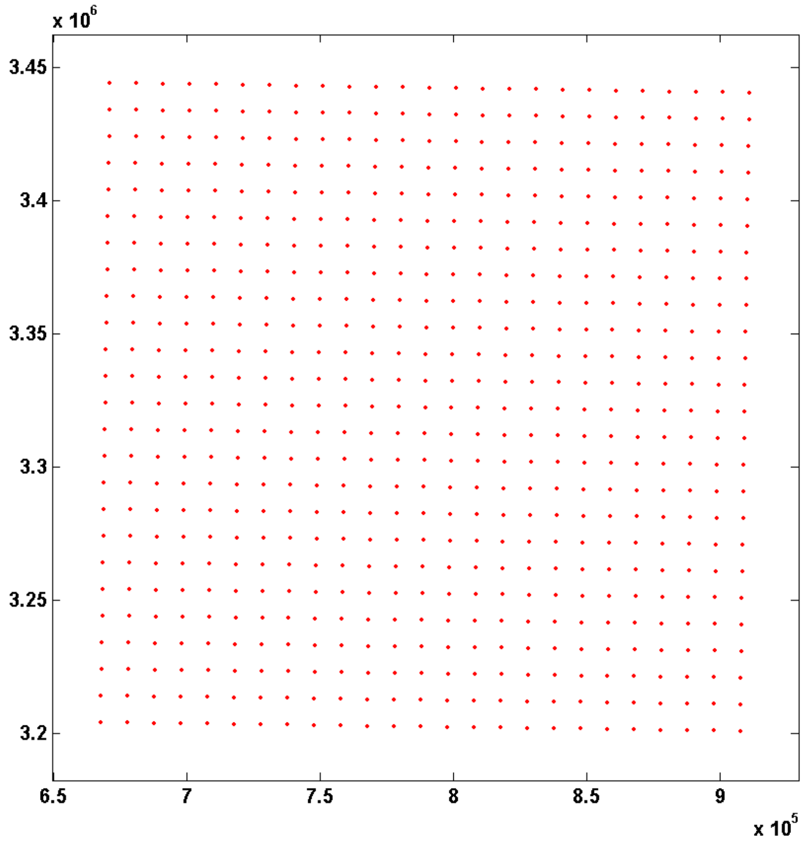


The Parallelogram Mistake in Interpretation Workstation





Segmented Quadrilateral Gridding



In segmented quadrilateral gridding the whole project polygon is subdivided (576 times in this case) and all vertices are reprojected cartographically. The bin centers are populated trigonometrically within each smaller quadrilateral.

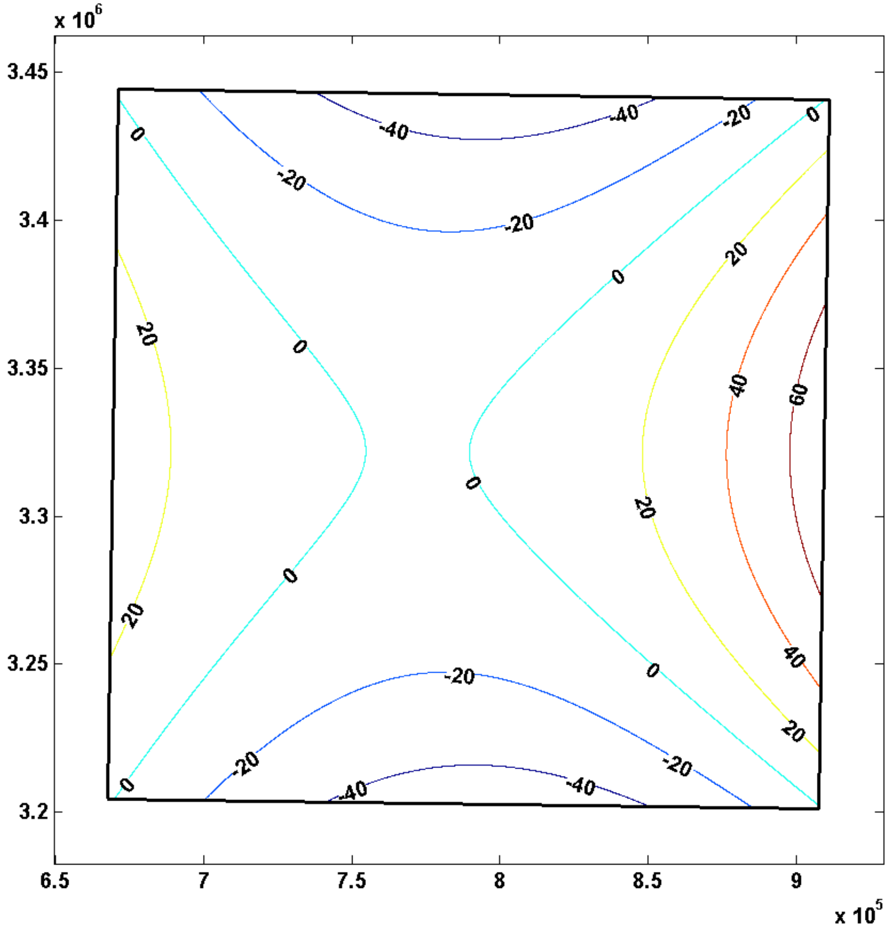
Efficiency and Accuracy Analysis

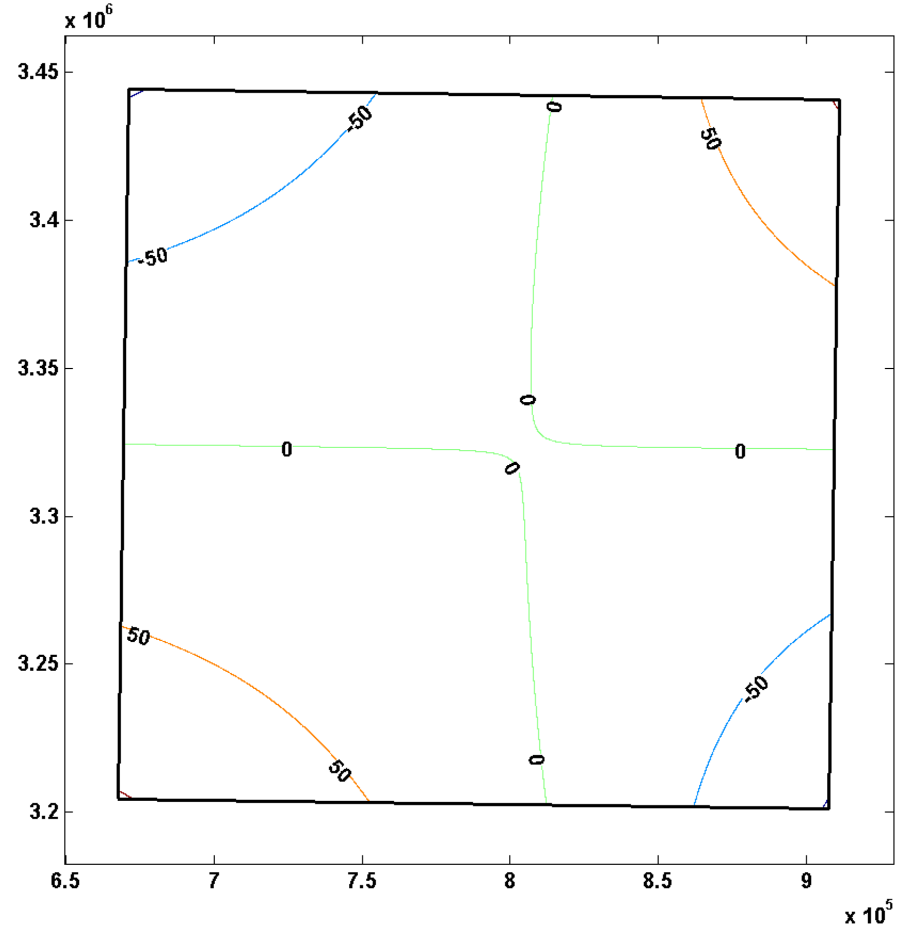
	Reprojection	**Workstation**	**Gridding**
Time	115.7 minutes	2.4 minutes	2.8 minutes
Accuracy	Reference	up to 103m	0.39m

- Project is 240km square
- Bin size is 100m square (5,764,801 points)
- Computer is Dell C610 (1.1GHz)
- Software is Matlab

"Best-Fit" Parallelogram Errors

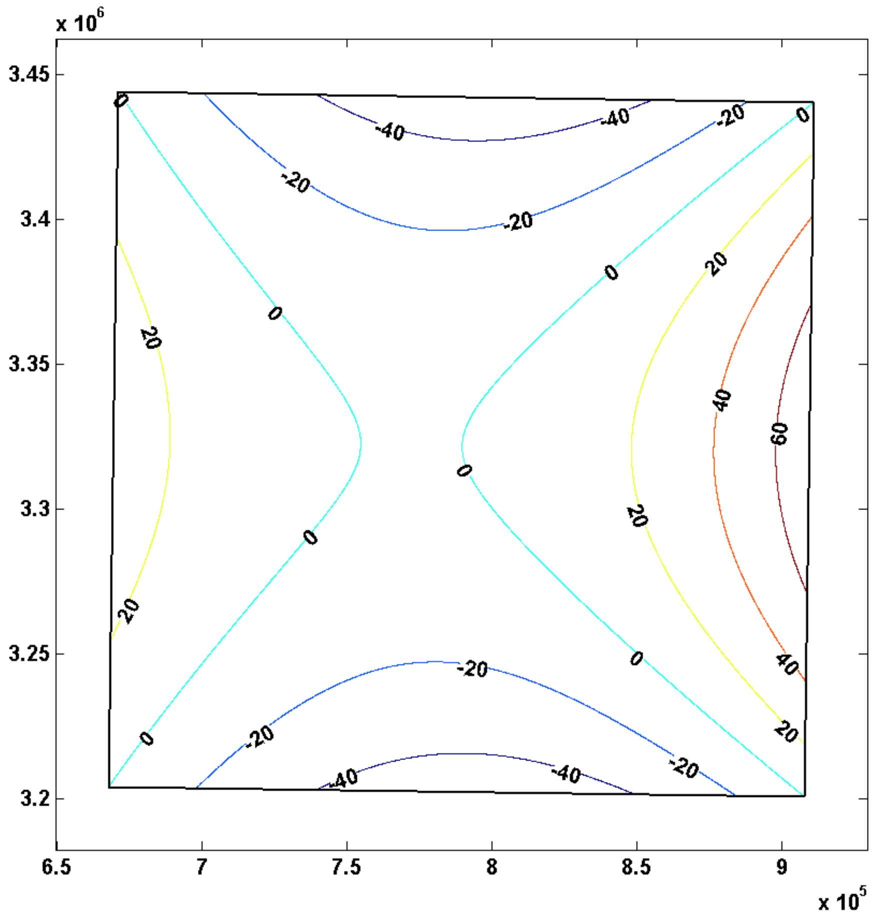
Easting (73m) Northing (103m)



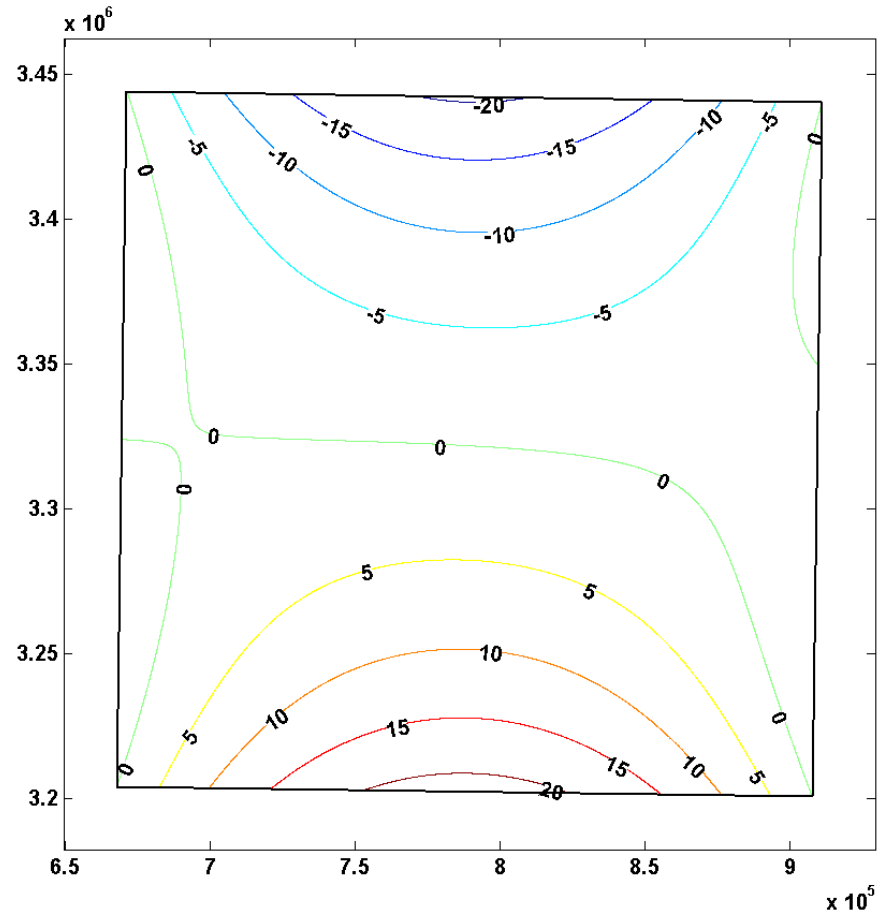


"Best-Fit" Quadrilateral Errors

Easting (73m)

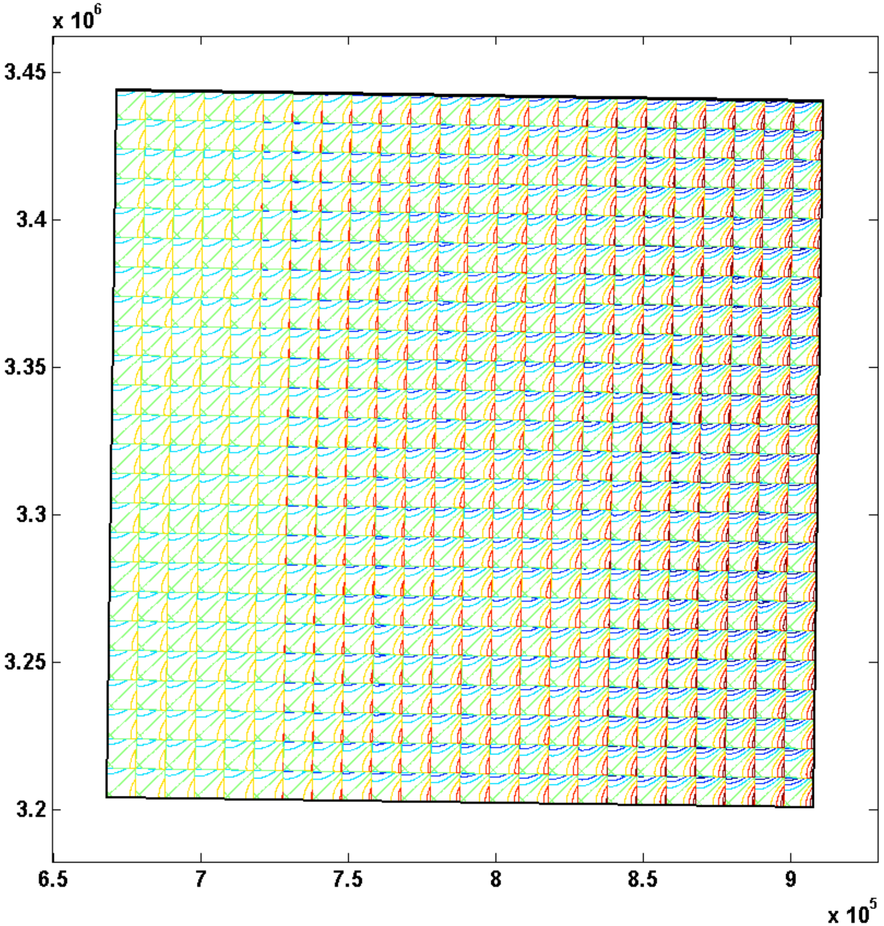


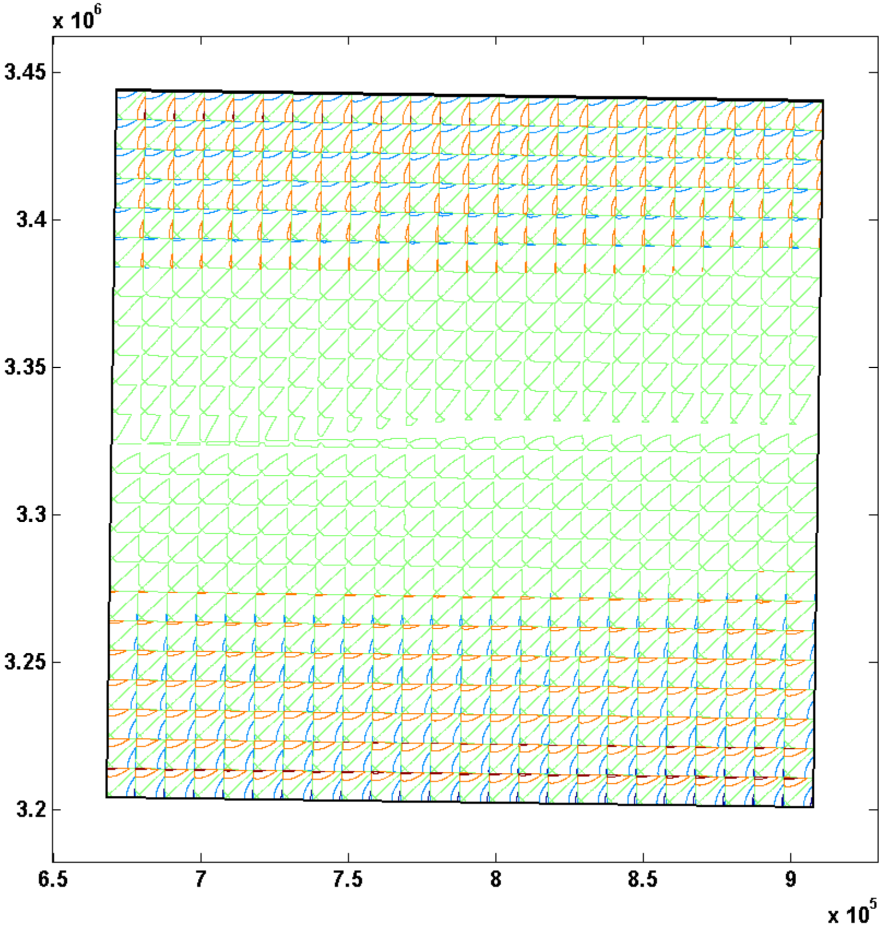
Northing (22m)



Segmented Quadrilateral Errors

Easting (0.39m) Northing (0.12m)





Efficiency and Accuracy Analysis

	Reprojection	**Workstation**	**Gridding**
Time	115.7 minutes	2.4 minutes	2.8 minutes
Accuracy	Reference	up to 103m	0.39m

- Workstation whole-project trigonometric loading is much faster than reprojecting every bin center (2.4 vs. 115.7 minutes), but inaccuracy is introduced (103m in this large project)
- Segmented quadrilateral gridding is also fast (2.8 vs. 2.4 minutes), but accuracy is 2 to 3 orders of magnitude better than the common workstation technique (0.39m)

Final Comments

- Whole projects reprojected cartographically (every bin center) from one conformal projection to another always preserve orthogonality (though straight lines may curve slightly)
- Trigonometric shortcuts on some interpretation workstations ignore this cartographic reality, thus inducing error
- Segmented quadrilateral gridding is a fast and accurate compromise that preserves orthogonality